General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MFP2

Unit Further Pure 2

Wednesday 9 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 (a) Show that

$$9\sinh x - \cosh x = 4e^{x} - 5e^{-x}$$ *(2 marks)*

(b) Given that

$$9\sinh x - \cosh x = 8$$

find the exact value of $\tanh x$. *(7 marks)*

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2 (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions. *(3 marks)*

(b) Use the method of differences to find

$$\sum_{r=1}^{48} \frac{1}{r(r+2)}$$

giving your answer as a rational number. *(5 marks)*

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3 Two loci, L_1 and L_2, in an Argand diagram are given by

$$L_1 : |z + 1 + 3i| = |z - 5 - 7i|$$

$$L_2 : \arg z = \frac{\pi}{4}$$

(a) Verify that the point represented by the complex number $2 + 2i$ is a point of intersection of L_1 and L_2. *(2 marks)*

(b) Sketch L_1 and L_2 on one Argand diagram. *(5 marks)*

(c) Shade on your Argand diagram the region satisfying

both $$|z + 1 + 3i| \leqslant |z - 5 - 7i|$$

and $$\frac{\pi}{4} \leqslant \arg z \leqslant \frac{\pi}{2}$$ *(2 marks)*

4 The roots of the cubic equation

$$z^3 - 2z^2 + pz + 10 = 0$$

are α, β and γ.

It is given that $\alpha^3 + \beta^3 + \gamma^3 = -4$.

(a) Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

(b) (i) Explain why $\alpha^3 - 2\alpha^2 + p\alpha + 10 = 0$. *(1 mark)*

(ii) Hence show that

$$\alpha^2 + \beta^2 + \gamma^2 = p + 13$$ *(4 marks)*

(iii) Deduce that $p = -3$. *(2 marks)*

(c) (i) Find the real root α of the cubic equation $z^3 - 2z^2 - 3z + 10 = 0$. *(2 marks)*

(ii) Find the values of β and γ. *(3 marks)*

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5 (a) Using the identities

$$\cosh^2 t - \sinh^2 t = 1, \quad \tanh t = \frac{\sinh t}{\cosh t} \quad \text{and} \quad \operatorname{sech} t = \frac{1}{\cosh t}$$

show that:

(i) $\tanh^2 t + \operatorname{sech}^2 t = 1$; *(2 marks)*

(ii) $\frac{\mathrm{d}}{\mathrm{d}t}(\tanh t) = \operatorname{sech}^2 t$; *(3 marks)*

(iii) $\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{sech} t) = -\operatorname{sech} t \tanh t$. *(3 marks)*

(b) A curve C is given parametrically by

$$x = \operatorname{sech} t, \quad y = 4 - \tanh t$$

(i) Show that the arc length, s, of C between the points where $t = 0$ and $t = \frac{1}{2}\ln 3$ is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, \mathrm{d}t$$ *(4 marks)*

(ii) Using the substitution $u = \mathrm{e}^t$, find the exact value of s. *(6 marks)*

Turn over ▶

6 (a) Show that $\dfrac{1}{(k+2)!} - \dfrac{k+1}{(k+3)!} = \dfrac{2}{(k+3)!}$. *(2 marks)*

(b) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$$

(6 marks)

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7 (a) (i) Express each of the numbers $1 + \sqrt{3}\,\mathrm{i}$ and $1 - \mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$. *(3 marks)*

(ii) Hence express

$$(1 + \sqrt{3}\,\mathrm{i})^8 (1 - \mathrm{i})^5$$

in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$. *(3 marks)*

(b) Solve the equation

$$z^3 = (1 + \sqrt{3}\,\mathrm{i})^8 (1 - \mathrm{i})^5$$

giving your answers in the form $a\sqrt{2}\,\mathrm{e}^{\mathrm{i}\theta}$, where a is a positive integer and $-\pi < \theta \leqslant \pi$. *(4 marks)*

END OF QUESTIONS